General Physics: Electromagnetism, Problem Set 11

Exercise 1 :

Consider an infinitely long, cylindrical conductor of radius R carrying a current I with a non-uniform current density

$$J = \alpha r$$

where α is a constant. Find the magnetic field everywhere.

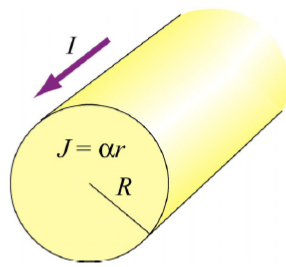


Figure 1: Non-uniform current density

Exercise 2 :

A circular loop of radius r_0 rotates with angular speed ω in a fixed magnetic field as shown in the Figure below.

1. Find an expression for the emf induced in the loop.
2. If the magnitude of the magnetic field is 25 μT, the radius of the loop is 1 cm, the resistance of the loop is 25 Ω and the rotation rate ω is 3 rad/s, what is the maximum current in the loop?

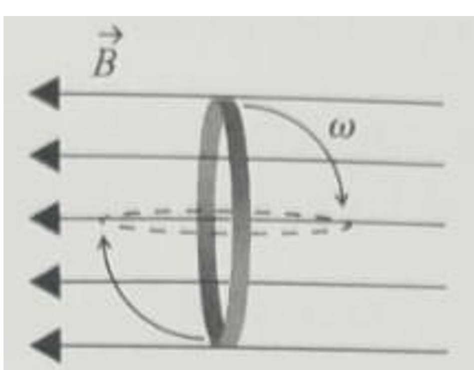


Figure 2: Circular loop rotating in a homogeneous magnetic field at an angular speed ω.

Exercise 3 :

Figure below shows a long solenoid with radius R and n turns per unit length; its current decreases with time according to

$$I(t) = I_0 e^{-\alpha t}.$$

1. What is the magnitude of the induced electric field at a point a distance r from the central axis of the solenoid when $r > R$ and $r < R$?
2. What is the direction of the induced field at both locations?
3. Sketch the function $E(r)$ for both $r < R$ and $r > R$

   Assume the infinite-solenoid approximation is valid throughout the regions of interest.

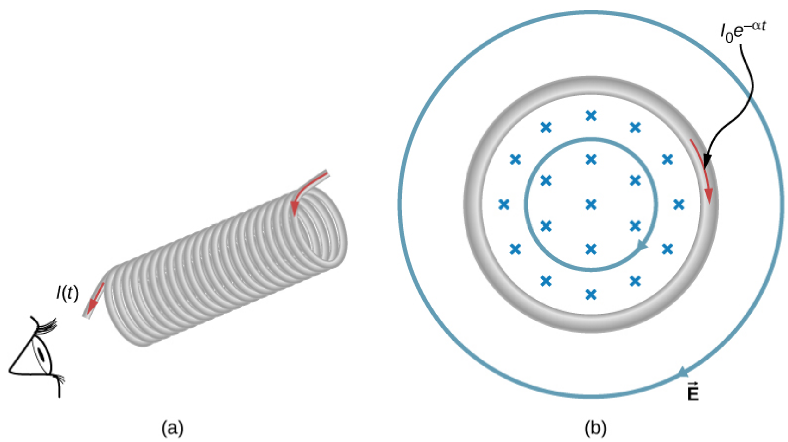
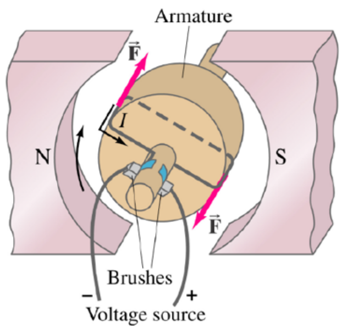


Figure 3: (a) The current in a long solenoid is decreasing exponentially. (b) A cross-sectional view of the solenoid from its left end. The cross-section shown is near the middle of the solenoid. An electric field is induced both inside and outside the solenoid

Exercise 4 :

A small electric car experiences a frictional force of 250 N when moving at a constant speed of 35 km/h. The electric motor is powered by ten batteries of 12 V each, connected in series. It contains a rectangular coil with 270 turns and of size 12×15 cm, which rotates in a magnetic field $B = 0.6$ T. The motor is directly connected (without gearbox) to the wheels with a diameter of 58 cm, so the coil and the wheels turn at the same frequency. We consider the moment when the coil is in the same plane as the magnetic field $\vec{B}$, such that the generated torque is maximal.

1. What is the current passing through the motor?
2. What is the emf induced in the coil?
3. What is the dissipated power in the coil?
4. What percentage of power produced by the motor is delivered to the wheels?



Exercise 5 :

A square conductive loop with side length $l = 10\,\mathrm{cm}$ and total resistance $R = 10\,\Omega$ is immersed in a uniform magnetic field $B = 0.52\,\mathrm{T}$. The loop is constrained to rotate about a fixed axis orthogonal to the direction of the field, as shown in the figure below. Let θ be the angle between the direction of the magnetic field and the normal to the loop. The loop is kept in rotation with a constant angular velocity $\omega = 12.4\,\mathrm{rad/s}$.

Determine:

(a) the absolute value of the electric charge passing through the loop during half a turn between the positions $\theta = 0$ and $\theta = \pi$;

(b) the external mechanical torque required to maintain the loop in rotation, and its average value over one turn;

(c) the work done by this mechanical torque over one turn. Remember that the work done by a torque τ is given by $W = \int_{\theta_1}^{\theta_2} \tau d\theta$, where θ_1 and θ_2 represent the initial and final angular positions.

(d) the energy dissipated in the loop over one turn.

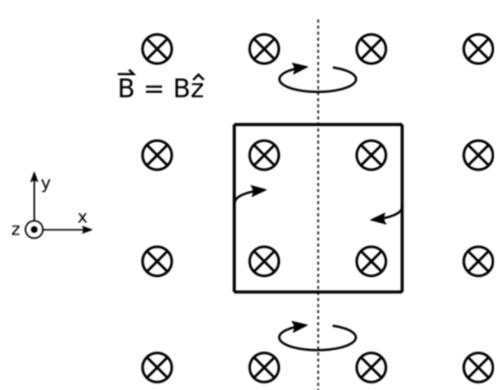


Figure 4: Square conducting loop in magnetic field